Virginia Math *Bulletin*

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Issue 5 Volume 1 Fall 2018

Views from the Chairs

Zoran Grujić
Professor of Mathematics
Interim Chair 2017-2018

Greetings friends, old and new. I am writing this after completing my one year-term as the Interim Chair, happy to report that the Department is alive and well!

The stage set by Craig Huneke's four-year term as the Chair was the one of a department in which the faculty and the students (graduate and undergraduate alike) were pursuing a variety of exciting avenues of the mathematical inquiry, a department committed to exploring new ways of how we teach mathematics (particularly Calculus) by adopting/redesigning a variety of student-centered, active learning methodologies, as well as a department fully engaged in outreach across the University, the Commonwealth and beyond; hence, it should come as no surprise that the academic year 2017-2018 was a productive one. A glimpse of what transpired can be gained from the rest of the newsletter, in what follows, I will just touch on several highlights.

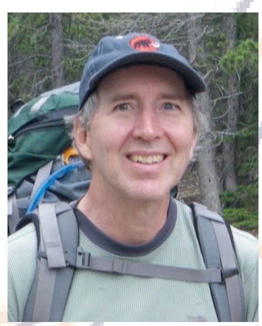
Our Calculus transformation efforts initiated by Craig and spearheaded by Paul Bourdon received a boost this year by hiring James Rolf who previously served as Director of Quantitative Learning at Yale.

A number of our faculty members received national honors and awards; in particular, Karen Parshall received The Albert Leon Whiteman Memorial Prize of the AMS, Julie Bergner received The Ruth I. Michler Memorial Prize of the AWM, Nicholas Kuhn and Weiqiang Wang were inducted in the 2018 Class of the Fellows of AMS, and John Imbrie and Andrei Rapinchuk held their Simons Fellowships in Mathematics.

On the undergraduate side, Grace Dwyer, Kevin Lee and Luca Scerbo were inducted in Phi Beta Kappa and Sebastian Haney was awarded a Goldwater Scholarship. On the graduate side, Mariano Echeverria and Veronica Shalotenko were the recipients of All-University Graduate Teaching Awards and Mike Reeks received Class of 1985 Fellowship for Creative Teaching.

The Institute of Mathematical Sciences (IMS) experienced a flurry of activity, including two series of Virginia Mathematics Lectures,by Yair Minsky (Yale) in the Fall and Irene Fonseca (Carnegie Mellon) in the Spring, two IMS Special Lectures by Barry Simon (Caltech), an IMS Public Lecture (co-sponsored with Physics) by Jacob Sherson (Aarhus) on citizen science, as well as two workshops. In addition, the public lectures sponsored by theA&S Diversity and Inclusion Grant, awarded to Sara Maloni, sparked the interest of the broader University community; the lectures were given by Moon Duchin (Tufts), Abby Stewart (Michigan) and Henry Segerman (Oklahoma State), addressing the topics of mathematics of gerrymandering, changing departmental culture to be more inclusive, and artistic mathematics, respectively.

On a personal note, I am quite ready to go back to research and teaching as my favorite pastimes. The good news is that John Imbrie is here ready to take our department to the exciting new adventures.



John Imbrie
Professor of Mathematics
Chair

I write this note as I begin a three-year term as chair. I am humbled by all that has been accomplished in the department during the five years since my interim chairmanship in 2012-3: four years under Craig Huneke's guidance, and last year with Zoran Grujic at the helm. There have been quite a few new faculty added over this timespan, and with these additions comes a feeling of anticipation and activity with the new term just around the corner. I know that we are all dedicated to making the department the best it can possibly be, by working as hard as we can to find excellence in new hires and new admissions, and by fostering a nurturing environment for newcomers and veterans alike.

The upcoming year will be a busy one, with a faculty search in the works, several promotions to attend to (a welcome and natural consequence of the hiring we have been doing), and continued expansion of calculus reform (with new faculty member James Rolf joining Paul Bourdon in that effort).

Readers of this newsletter series will see over the last few issues all the momentous changes and accomplishments over these years. My heartfelt thanks goes to Zoran for accepting the chair position during the last year. During this time, I was able to pursue a year of research leave in Princeton, developing exciting ideas and new collaborations that will keep me busy during lulls in department business -- assuming there are any! His dedication to the department's well-being has been evident from the beginning. I realize that every challenge he faced and surmounted during the last year makes my work that much easier. I only hope I can serve the department as well as Craig and Zoran have. With the efforts of our dedicated faculty, staff, and students, and with the support of the Dean's office and alumni, I have every expectation for another banner year in Kerchof Hall.

Supporting Us

The Mathematics Department is grateful for the generous support of its alumni and friends. The Department welcomes gifts annually to address its most urgent needs, as well as to the endowment which provides funding in perpetuity. To learn about how you can make a difference by supporting the Mathematics Department, please contact Liz Blaine at lblaine@virginia.edu or (434) 924-6156. To make a gift online, please visit http://giving.virginia.edu/mathematics

New Faculty Profiles

Benjamin Hayes
Assistant Professor
(2017-)

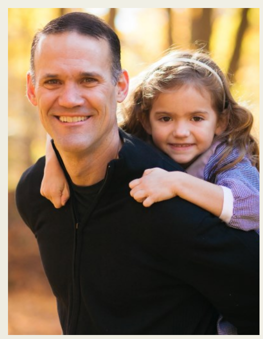
Working with colleagues in the Department of Mathematics' operator theory group, Benjamin Hayes researches topics involving the measurement of how many finitary approximations there are of a given infinitary object, including: entropy for actions of nonamenable groups, free probability with connections to von Neumann algebras and random matrices, and sofic groups.

Hayes arrives at the University of Virginia with a grant from the National Science Foundation's Division of Mathematical Sciences for his continuing research. He has published nine papers, including articles in Geometric and Functional Analysis, International Mathematics Research Notices and Journal of the Institute of Mathematics Jussieu. As a graduate student at UCLA, Hayes earned a Dissertation Year Fellowship and the Heaviside Wealth Management Award, which recognizes the graduate student who does the best job explaining their research to someone outside of their field.

Hayes received his B.S. in mathematics from the University of Washington (2009) and earned his Ph.D. in mathematics from UCLA (2014).

Hayes will be teaching an Introductory Real Analysis course in the fall and Calculus on Manifolds in the spring. Excited to work with the department's operator theory group, Hayes also plans to explore possible collaborations and connections with the department's algebra and probability groups.

Jim Rolf
Associate Professor
(2018-)



Trained as a numerical analyst, James (Jim) Rolf currently focuses on the scholarship of teaching and learning mathematics. He has published a number of articles and is a co-author of the book *Explorations in Complex Analysis* (Mathematical Association of America, 2012). His work has been supported by numerous grants funded by the Association of American Universities, the National Science Foundation; Microsoft Corporation, Yale's Center for Teaching and Learning, and others.

Rolf received a Ph.D. from Duke University in 1997, a Master of Divinity from Southwestern Theological Seminary, and a bachelor's of science from Baylor University. Before coming to the University of Virginia, he served as the Shizuo Kakutani Lecturer at Yale University, as an associate professor of mathematics at the United States Air Force Academy and as an assistant professor at the United States Military Academy at West Point.

Joining the Department of Mathematics, Rolf will teach Math 1320 (Integral Calculus) this next year and will help design metrics to assess changes in student learning and attitudes in this course when changes to course design are implemented.

He also will continue to work with other institutions as they seek to implement and assess innovative ways to help incoming students prepare for quantitative reasoning requirements at their institution.

Vivek Mukundan
Postdoctoral Fellow
(2017-)

Working in the areas of commutative algebra and algebraic geometry, Vivek Mukundan's research spans methods for computing the defining ideal of the Rees algebra, studying the invariants of powers of edge ideals, multiplicity theory, koszul algebras and other topics.

Vivek received his master's degree in mathematics from the Indian Institute of Technology in Madras, India, and a Ph.D. in mathematics from Purdue University. Before coming to the University of Virginia, he was a visiting fellow at the Tata Institute of Fundamental Research in Mumbai, India.

Mukundan's research grants and fellowships include multiple summer research grants from Purdue University, a National Board for Higher Mathematics postdoctoral fellowship from India's Department of Atomic Energy, an INSPIRE Faculty Award from India's Ministry of Science & Technology, a Jawaharlal Nehru Centre for Advanced Scientific Research Fellowship, and travel grants from the American Mathematical Society and the Mathematical Research Communities program.

At UVA, he plans on furthering his research in the fields of commutative algebra and algebraic geometry while expanding his teaching repertoire.

New Faculty Profiles

Prasit Bhattacharya
Whyburn Instructor
(2017-)

Working in the area of algebraic topology, Prasit Bhattacharya researches computational aspects of stable homotopy theory. Specifically, he explores stable homotopy groups of spheres, using chromatic homotopy theory. He studies vn self-maps that result in infinite families of elements in stable homotopy groups of spheres, and his current research involves C 2- equivariant computations, with a focus on the telescope conjecture.

Bhattacharya completed his bachelor's degree (2007) and his master's degree in mathematics (2009) at the Indian Statistical Institute in Bangalore, India. He completed his Ph.D. at Indiana University (2015). Bhattacharya comes to the University of Virginia from the University of Notre Dame, where he served as a visiting assistant professor (2015-2017).

Bhattacharya has taught mathematics courses at all college levels, including pre-calculus, calculus (at various levels), linear algebra and finite mathematics. He enjoys mentoring undergraduate students as well as high-school students. Bhattacharya hopes to continue mentoring students at UVA and to teach courses at various levels while organizing graduate-level seminars, as he has at previous institutions.

Liron Speyer
Whyburn Instructor
(2017-)

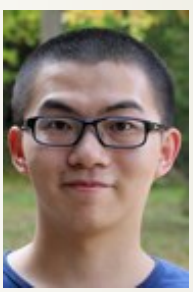
A postdoctoral researcher specializing in representation theory, Liron Speyer focuses his work on the study of a fundamental object known as the symmetric group, as well as several families of related mathematical objects. These include the quiver Hecke algebras introduced in the last decade, which have brought about a surge of interest in the area.

Liron will be joining the University of Virginia directly from Osaka University, Japan, where he held a postdoctoral fellowship funded by the Japan Society for the Promotion of Science. After receiving his Ph.D. in mathematical sciences from Queen Mary University of London, and his master's degree in mathematics from the University of Warwick, he held a visiting postdoctoral position at the University of East Anglia, funded by the London Mathematical Society.

Liron's work has been published in Transactions of The American Mathematical Society, Proceedings of The American Mathematical Society, International Mathematics Research Notices, as well as three top algebra journals.

This academic year, Liron will teach algebra courses for science majors, while his research will largely focus on constructing a vast generalization of the famous Littlewood –Richardson Rule in the context of quiver Hecke algebras.

Jinbo Ren
Postdoctoral Research Associate
(2018-)



Working in the areas of number theory and algebraic geometry, Jinbo Ren's research mainly concerns subjects on the arithmetic and geometric properties of abelian varieties and Shimura varieties, for example the André-Oort and Zilber-Pink conjectures. He also works in related topics in symmetric spaces, transcendental number theory, model theory, and group theory.

Jinbo received his master's degree in mathematics from both Universiteit Leiden, the Netherlands and Université Paris-Sud, France. He completed his Ph.D. in mathematics from Université Paris-Sud, France. Before coming to the University of Virginia, he spent three years at Institut des Hautes Études Scientifiques (IHES), France.

Besides the pursuit of excellent mathematical research, Jinbo has a great passion for teaching. For him, each day with lectures is a holiday. He radiates an infectious enthusiasm in making everyone in his class enjoy the beauty of mathematics, instead of only coping with exams. Jinbo also takes classical literature as a pastime.

Mouhamadou Sy
Postdoctoral Research Associate and Whyburn Lecturer
(2017-)

Mouhamadou Sy attended high school in Mauritania, then joined the University of Cergy-Pontoise in France for his undergraduate and graduate studies.

In 2017, he received his PhD from the University of Cergy-Pontoise. His current research interests include Partial Differential Equations and Mathematical Physics. Recently, his work centers on his interest in the description of longtime behavior of some models that arise from fluid dynamics and relativistic quantum mechanics via the theory of invariant measures. This method employs statistical equilibria of systems to predict the individual behavior of their constitutive particles.

He also likes working with students in order to introduce them to different mathematical concepts and scientific thought. During his graduate studies at UCP, he supervised undergraduate students working on their final dissertations. In 2015, he received an award in recognition of this work from the FSPM (Mathematical Sciences Foundation of Paris), in collaboration with the Casden Bank.

At UVa, he plans to develop probabilistic methods to approach the Cauchy problem of supercritical equations - an outstanding open problem in the theory of Partial Differential Equations.

Faculty Awards and Honors

KAREN PARSHALL RECEIVES 2018 AMS WHITEMAN PRIZE

Commonwealth Professor of Mathematics and History Karen Hunger Parshall received the 2018 Albert Leon Whiteman Memorial Prize of the AMS for her outstanding work in the history of mathematics, and in particular, for her work on the evolution of mathematics in the USA and on the history of algebra, as well as for her substantial contribution to the international life of her discipline through students, editorial work, and conferences.

The Albert Leon Whiteman Memorial Prize recognizes notable exposition and exceptional scholarship in the history of mathematics. It was established in 1998 using funds donated by Mrs. Sally Whiteman in memory of her husband, Albert Leon Whiteman. The 2018 prize was awarded Thursday, January 11, 2018, at the Joint Mathematics Meetings in San Diego.

JULIE BERGNER RECEIVES MICHLER MEMORIAL PRIZE

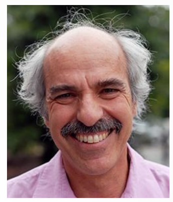
Associate Professor Julie Bergner has been selected as the winner of the 2018-2019 Ruth I. Michler Memorial Prize competition.

The Ruth I. Michler Memorial Prize of the AWM is awarded annually to a woman recently promoted to Associate Professor or an equivalent position in the mathematical sciences. The Ruth I. Michler Memorial Prize honors outstanding women at this stage of their careers and enable them to focus on their research in the stimulating environment of the Cornell University Mathematics Department.

A USD 47,000 prize will be awarded to a woman, recently promoted to associate professor or the equivalent, for a semester of mathematical research without teaching obligations in the Mathematics Department of Cornell University. A supplement housing/subsistence stipend award of USD 3,000 will be provided. Office space, library access, and computing facilities will be provided by Cornell. The award is to be used during the 2018-2019 academic year.

She was selected for her "proposed project to connect some of her recent work with the research of Cornell faculty member Inna Zakharevich, including simultaneous developments by both women (and their respective coauthors) on algebraic K-theory constructions." Bergner's research has been in the areas of homotopy theory. Her proposed research will bring together several facets of her work: the theoretical framework of homotopical categories and generalizations, the realization of 2-Segal spaces as a form of algebraic K-theory, and looking at derived Hall algebras as algebraic homotopical categories.

NICHOLAS KUHN AND WEIQIANG WANG NAMED 2018 AMS FELLOWS



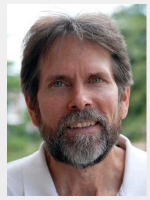
Professors Nicholas J. Kuhn and Weiqiang Wang have been named 2018 Fellows of the American Mathematical Society:

Nicholas Kuhn "For contributions to homotopy theory, group cohomology and representation theory."

Weiqiang Wang "For contributions to Lie theory and representation theory and service to the mathematical community."

Sixty-three mathematical scientists have been named 2018 Fellows of the AMS. AMS President Kenneth A. Ribet says, "This year's class of AMS Fellows has been selected from a large and deep pool of superb candidates. It is my pleasure and honor as AMS President to congratulate the new Fellows for their diverse contributions to the mathematical sciences and to the mathematics profession."

PAUL BOURDON PROMOTED TO FULL PROFESSOR



Paul Bourdon was promoted to Professor of Mathematics, General Faculty this year. He came to UVa in 2013 as a Visiting Professor of Mathematics, having held appointments at Michigan State University, the University of Tennessee at Knoxville, and Washington & Lee University where he was Cincinnati Professor of Mathematics. Bourdon's research interests are operator theory (especially composition operators acting on spaces of analytic functions), function theory (analytic & harmonic), and quantum-information theory (superdense coding with partially entangled quantum particles). His research has been supported through four grants and two research-opportunity awards from the National Science Foundation. Bourdon received, April 2000, the John Smith Award for Distinguished Teaching of Mathematics, conferred by the MD-VA-DC Section of the Mathematical Association of America. He is currently Director of Lower Division Courses for the Mathematics Department.

BEN HAYES AWARDED EXCELLENCE IN POSTDOCTORAL RESEARCH PRIZE

Assistant Professor of Mathematics Ben Hayes was awarded the Amir Aldroubi and Amira Azhari Prize for Excellence in Postdoctoral Research from Vanderbilt University on Wednesday, April 18, 2018.

The Samir Aldroubi and Amira Azhari Prize for Excellence in Postdoctoral Research is awarded every two years to recognize the research achievements of current and recent postdoctoral fellows in the department. It was established by Mathematics Professor Akram Aldroubi in honor of his parents, Samir Aldroubi and Amira Azhari. Candidates for the Aldroubi-Azhari Award are nominated by the faculty of the department, and the recipient is selected by a departmental committee. The chosen mathematician receives $1,000 and is invited to Vanderbilt to deliver a mathematics colloquium reflecting his or her research interests.

Transforming Calculus

The course was challenging, but I learned a lot of new material in ways that were easier to understand and implement. I liked the "flipped" aspect of the class because I find that doing problems is the best way to learn mathematics. Overall, the course went well and I learned a lot.

- Comment from a student's course evaluation of "transformed" Calculus I (Math 1310)

By Paul Bourdon

In October of 2016, Mathematics-Department faculty approved a proposal to transform calculus instruction at UVa by reducing class sizes, increasing the amount of time students spend in class engaged in problem-solving activities—especially group-based ones, and augmenting learning objectives for the course to include improvement of technical reading and writing skills as well as development of confidence and competence in communicating technical information orally.

The "Transforming Calculus" proposal, authored by Craig Huneke (Chair of the Department, 2013-2017), also calls for primary instructors to conduct all class meetings (those labeled "discussion" as well as those labeled "lecture"), adding a fourth hour of instruction in 1200- level calculus courses, creating new general-faculty and postdoctoral positions to provide additional instructors needed to support smaller class sizes and more hours of instruction per week, and expanding the Math Tutoring Center to increase support for calculus students outside of the classroom.

Some Background

The Department offers a two-tier system of calculus instruction: a one-year sequence (1210–1220, A Survey of Calculus I–II) for students interested primarily in the life, managerial, and social sciences, and a one-year sequence (1310–1320, Calculus I for students interested primarily in the natural sciences (including mathematics). In the Spring and Fall of 2015, 1742 students took 1210 or 1220 in 44 sections. During the same period, 514 students took 1310 or 1320 in 17 sections. A significant portion of all students in the College pass through these courses. Each course in the 1300-level sequence carries four credits, providing 200 minutes of class time per week, while currently each course in the 1200-level sequence carries three credits, providing 150 minutes of instruction per week.

Instructor Jon Simone responds to a student's question during Calculus-I groupwork.

Implementation

The "Transforming Calculus" proposal calls for modifying courses gradually over a five-year period, with the "science track" Calculus-I course (Math 1310) the first to be transformed. During the Fall term of the 2016–2017 academic year, Paul Bourdon (Math Department), Gail Hunger (Learning De- sign and Technology), Peter Bonventre (Math GTA), and Katelynn Kochalski (Math GTA) met weekly to design a transformed Calculus-I course, based on a flipped-classroom, groupwork-intensive model of instruction used successfully at the University of Michigan. The structure of "Transformed Calculus I" is described as follows on the current syllabus for the course:

All sections of Math 1310 are based on active- and cooperative-learning strategies designed to further develop your problem-solving skills applicable in any situation.

During our Tuesday and Thursday class meetings, at least 70% of the time you will be engaged in groupwork with your classmates; the rest of the time will be devoted to mini-lectures (by me), problem-solution discussions (led by students), and whole-class discussions of concepts, techniques, and problem-solving principles. During our final class meetings on Fridays, we will review topics from the first two class meetings of the week and typically have a quiz on those topics. For our Tuesday and Thursday class meetings, you'll be expected to familiarize yourself—through on-line class-prep assignments (accompanied by a video)—with the basic notions and ideas that will play a role in class.

The design of this course is based on research showing:

Students learn best when they take an active role

- When they discuss what they are reading
- When they practice what they are learning
- When they apply practices and ideas.[1]

The first sections of Math 1310 having the structure described above were offered Spring '17. Four of eight sections offered Fall '17 were taught in the transformed, flipped -classroom style, and, starting this fall, all sections offered will be flipped. Spring 2019, Calculus II (Math 1320) classes will be transformed, with implementation directed by Jim Rolf, a newly hired general-faculty member, formerly the director of quantitative learning at Yale University. Transformation of Math 1220, A Survey of Calculus II, is scheduled for implementation fall of 2020; and of Math 1210, A Survey of Calculus I, fall of 2021.

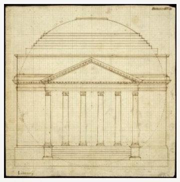
Assessment.

Our assessments of the effectiveness of the format of the transformed Math 1310 course are consistent with research indicating that students learn more when they engage in collaborative problem solving with their peers during class. For example, Fall '17, students in flipped sections of Math 1310, on average, achieved normalized gains on the "Calculus Concept Inventory" 11% higher than those of students in traditionally taught sections and scored between 5.9% and 38.2% higher on multiple-choice assessment -problems on the common Math 1310 final exam. Moreover, in their course evaluations, students in flipped sections indicated greater agreement with statements such as "I learned a great deal in this course," and many praised the structure of the course.

1 From Tools of Teaching by B.G. Davis, Jossey-Brass, San Francisco 1993.

Department Activities

CIRMATH-AMERICAS CONFERENCE



A conference on the role of journals in mathematics was hosted by Professor Karen Parshall and sponsored by IMS.

CIRMATH includes researchers from Europe, the United States, Canada, and Latin America. Since 2014, it has sponsored a series of colloquia at the Institut Henri Poincaré in Paris and at the University of Lorraine., while various of its members have participated in conferences at the Royal Society of London, the Lisbon meeting of the European Society for the History of Science, the Centro internazionale per la ricerca matematica in Trento, Italy, the Fourth International Conference on the History of Mathematics Education in Turin, Italy, and the Mittag-Leffler Institute in Stockholm. The CIRMATH-Americas meeting at the University of Virginia was the group's first meeting in the United States, a country which - especially in the period from 1850 to 1950 - played an increasingly large role in international mathematical developments.

The CIRMATH-Americas meeting has the following goals:

To provide a venue in which European CIRMATH members can intensively interact with historians of mathematics from the Americas working on related historical questions.

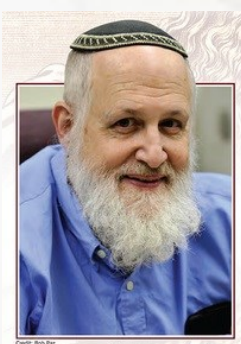
To provide researchers the opportunity to explore the significant archival and library holdings at both the University of Virginia and the nearby Library of Congress.

To provide opportunities for American and European researchers at earlier stages in their careers to develop significant and enduring research collaborations with established scholars in the history of mathematics.

To trace the impact of journals on the circulation of mathematics within the Americas and between the Americas and Europe from 1850 to 1950.

IMS Special Lectures

Barry Simon: Tales of our Forefathers



The UVa Department of Mathematics & The Institute of Mathematical Sciences presented two Special Lectures by Barry Simon (California Institute of Technology). The first of these, entitled "Tales of our Forefathers," took place on February 26th, with "More Tales of our Forefathers" occurring on the 28th, discussing Riemann, Newton, Krein, and Noether, among other mathematicians.

Simon described the lecture as "not a mathematics talk, but...a talk for mathematicians." He argued that "too often, we think of historical mathematicians as only names assigned to theorems. With vignettes and anecdotes," he continued, "I'll convince you they were also human beings and that, as the Chinese say, 'May you live in interesting times' really is a curse."

Jacob Sherson on Citizen Science: Benchmarking Human and Artificial Intelligence

On April 19th, Jacob Sherson (Department of Physics and Astronomy, Aarhus University, Denmark and ScienceAtHome) delivered a public lecture entitled "Citizen Science: Benchmarking Human and Artificial Intelligence," co-sponsored by the IMS and Departments of Mathematics and Physics.

To use Sherson's own words: "Artificial intelligence (AI) is a field is in rapid development and there are almost-daily news reports about how AI has revolutionized yet another industrial domain. Some researchers claim that within a few decades we will reach a so-called singularity in which computer intelligence will surpass human capabilities in all domains. Other AI-researchers, however, maintain that we have still far from understood the human ability to reach fast, intuitive and correct decisions based on often seemingly too little data. In the www.scienceathome.org project, we have developed games allowing so far 250,000 players to contribute to research in quantum and classical physics, mathematics, chemistry, behavioral economics, corporate management, psychology and cognitive science. We believe that this wealth of data from human individual and collective problem solving can be used to generate novel insights about human intuition and innovation that could potentially form the basis of novel forms of human-inspired AI.

Sherson concluded the talk with a description of how the work is situated "within the new global educational movement, Think Like a Scientist, in which we introduce citizen science games to the formal school setting at all levels as a means to make the world of research and knowledge generation available to the students from an early age.“

Virginia Mathematics Lecture Series

The Virginia Mathematics Lectures are a Distinguished Lecture Series that IMS established in 2014

Spotlight: Yair Minsky

Yair Minsky of Yale University was our guest in mid-November and delivered three lectures as part of the Virginia Mathematics Lectures series.

The first lecture, "Hyperbolic 3-manifolds, their structure and deformations," took place on November 13th, 2017. Minsky describes hyperbolic geometry as the "richest and most interesting of Thurston's eight geometries for 3-manifolds," explaining that "a good understanding of the ways in which hyperbolic geometry interacts with topology in three dimensions also informs our understanding of many related fields, such as geometric group theory and complex dynamics." The lecture served to provide "a brief introduction to this subject," with a focus on "examples, geometric intuition and overall structure.

The second lecture took place the following day, and was entitled "Between 2 and 3 dimensions: Teichmuller theory, pleated surfaces, and the complex of curves."

"One can probe the geometry of a 3-manifold," he continues, "by mapping in surfaces in different ways: conformal boundaries at infinity give us a parametrization of families of 3-manifolds using classical Teichmuller space, and Thurston's pleated surfaces relate the varying geometry on the interior to 2-dimensional combinatorial and geometric structures." The lecture explored these ideas before moving on to a brief explanation of "how they provide a complete set of invariants for deformation spaces of 3-manifolds."

The final lecture, "Quantitative models, Thurston's skinning map, and beyond," occurred on November 15th. "While the theory has had many successes," Minsky concludes, "we are still far from having a complete 'effective' recipe for predicting the geometry of a hyperbolic 3-manifold from its topological description." Minsky then highlighted some current work being done in the area, as well as the unanswered questions that future work will need to address.

Spotlight: Irene Fonseca

On March 15th and 16th, the Department was pleased to host Irene Fonseca of Carnegie Mellon University, who gave two talks on "Variational Methods for Materials and Imaging Sciences" as part of the Virginia Mathematics Lectures series.

Fonseca explains that "recently developed methods and a deep articulation of ideas from the Calculus of Variations, Geometric Measure Theory, and nonlinear Partial Differential Equations (PDEs), have been instrumental in the mathematical rigorous understanding of nonlinear phenomena in a plethora of physical and technological applications, ranging from analyzing instabilities in novel advanced materials to the denoising of medical images."

Undergraduate Experiences: AWM Lunch with Irene Fonseca

By Mia Shaker

On March 15, the AWM chapter of the UVA Math Department hosted a lunch with the Professor Irene Fonseca from Carnegie Mellon University. Professor Fonseca is a renowned mathematician, the director of Carnegie Mellon's Center for Nonlinear Analysis. She came to UVA to give a series of lectures on "Variational Methods for Materials and Imaging Sciences" in our ``Virginia Mathematics Lecture Series" sponsored by the Institute for Mathematical Sciences. She also kindly agreed to meet with a group of the AWM members as well as the visiting admitted mathematics graduate students.

At the beginning of the event, Irene Fonseca shared a little bit about her research interests, and she heard from everyone in attendance about what their areas of interest in mathematics are.

But the real focus of the lunch and discussion was on the representation of women in mathematics. Irene Fonseca talked about her experience as the only female professor in the CMU math department, and how that has been true for almost the entirety of her 30 years teaching. It opened up a larger conversation about what the admitted graduate students had seen in regards to representation in their own schools.

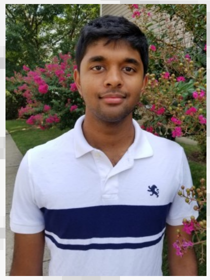
Another interesting dimension to the issue of women in mathematics that Irene Fonseca raised is the "two-body problem," which refers to how spouses in academia often struggle to obtain jobs at the same university, or within commuting distance of each other. Fonseca, while having seen the two-body problem stop several women she knows from getting the best job available to them, felt that CMU originally hired her in large part because they were hiring her then-husband.

As an undergraduate in the AWM, I often focus on the representation of women among students, but this lunch really broadened my perspective and had me looking towards the future. I found myself inspired by the way Irene Fonseca and other like her have been striving to succeed in such a male-dominated field.

Undergraduate Accomplishments

Arun Kannan Becomes First Graduate of the Distinguished Major Program in Mathematics

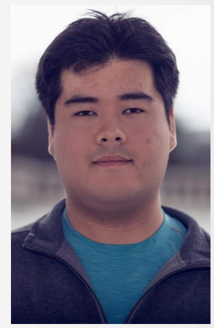
By Andrei Rapinchuk



The 2017-18 academic year saw the department roll out the revamped Distinguished Major Program (DMP). The program provides an opportunity for math majors, typically during their 4th year, to work on a research project under the supervision of a faculty member. The findings are then organized in the Distinguished Major Thesis and presented at a public defense.

The first graduate of the program was Arun Kannan who wrote his thesis, entitled *Characters of Projective Modules in the BGG category O for General Linear Lie Superalgebras*, under the supervision of Professor Weiqiang Wang during his 3rd year. While working on the thesis, Arun had developed the understanding of representation theory of semisimple Lie algebras and superalgebras at the level of second year PhD students. Arun successfully defended his thesis on April 4, 2018 and was presented with the DMP Completion Certificate at the Math majors dinner. In his 4th year, Arun will pursue a Master's Degree in mathematics.

Sebastian Haney Awarded Goldwater Scholarship



Mathematics major Sebastian Haney, currently in his third year, was the one of about 20 students in the nation selected to receive the 2018 Goldwater Scholarship.

The Goldwater Foundation awards the scholarships, which Congress established in 1986 to perennially honor the lifetime work of Senator Barry Goldwater, to college sophomores and juniors who "intend to pursue research careers in the natural sciences, mathematics, and engineering."

Gordon E. Keller Mathematics Majors Dinner

E. J. McShane Prize in Mathematics

April 2018

The 2018 E. J. McShane Prize in Mathematics was given to Stephen Davis and Arun Kannan for their achievements in mathematics.

2018 E.J. McShane Prize winners

2018 Floyd Prize Winners

Edwin E. Floyd Prize in Mathematics

April 2018

The 2018 Edwin E. Floyd Prize in Mathematics was awarded to Sebastian Haney, Benjamin Keigwin, Yichen Ma, and Ethan Zell. The prize is awarded to second- or third-year students who show exceptional promise in mathematics.

Speaker: Angus Mitchell

The 2018 Gordon E. Keller Mathematics Majors Dinner, supported by a generous gift by Doug and Laurel Costa, took place last Spring. This year featured Angus Mitchell as our guest speaker, who lectured about data analytics. Angus graduated from UVA in 2013 with a degree in Financial Mathematics, as well as an interdisciplinary degree in Computer Science. He has had a dynamic career in the intervening years, including work with the NBA .

Three Majors elected into Phi Beta Kappa

Congratulations to Grace Dwyer, Kevin Lee, and Luca Scerbo! As the oldest and most distinguished honor society in the country, Phi Beta Kappa offers membership to less than one percent of all undergraduates. Many of the leading figures in American history and culture have begun their careers with election to the society, including seventeen presidents of the United States. As a result, membership is a remarkable accomplishment, both for the student who achieves it and the faculty and staff whose support and guidance has led to this milestone.

William Lowell Putnam Mathematical Competition Award

April 2018

Ira Herbst with 2018 Putnam Award winners

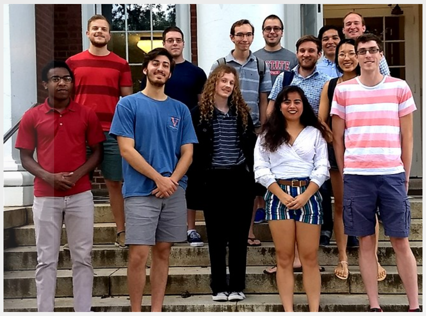
The 2018 William Lowell Putnam Mathematical Competition Award was given to Emmett Dorlester, Zachary Baugher, and Luca Scerbo for their outstanding scores on the exam.

SUMMER RESEARCH PROGRAM FOR UNDERGRADUATES

By AXEL SAENZ

This summer, the Math department was excited and pleased as it hosted, for the first time, a two-month summer Research Program for Undergraduates beginning in the middle of May and ending in the middle of July. The program was organized by the strong efforts and admirable vision of Associate Professor Julie Bergner and Associate Professor Thomas Mark. The participants included a total of seven undergraduate students: Cedric Harper, Trent Lucas, Fernanda Yepez-Lopez, and Ethan Zell from the University of Virginia; Eric Keener from James Madison University; and Connor Mallin from the University of Alabama.

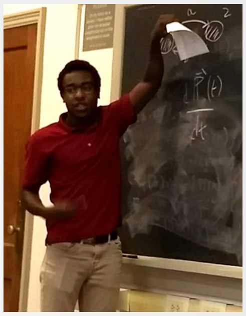
Fernanda Yepez=Lopez Presents to Group



Program Participants and Organizers

The summer program was intended to bridge the gap between textbook mathematics and research-level mathematics showing the students that the subject is still evolving through an immersive first-hand research experience. Moreover, there was a strong emphasis in developing the communication skills of the students as it is often the case that the seemingly obtuse and technical language of research-level mathematics prevents beginners to express their ideas clearly. To this effect, the students were introduced to selected research problems in mathematics apt to their diverse academic backgrounds and were asked to tackle these problems through attentive advising of UVa math graduate students - Rostislav Akhmechet, Zachary Gates, Andrew Kobin, Mark Schrecengost, and Richard Vrandenburgh- and postdocs - Prasit Bhattacharya and Axel Saenz Rodriguez.

The program began with a one-week series of lectures and exercises led by the graduate students and postdocs. At the end of the first week, the students chose and were selected to create two research groups focusing on topics in group theory and probability theory, respectively. From then on out, the groups would convene on a daily basis in the common areas of Kerchof Hall to carry out their research projects. This included independent readings, meetings with graduate and postdoc advisors, frequent presentations on the students' progress, and weekly talks related to the research projects presented by faculty, postdocs, and graduate students. Throughout the summer, both the students and advisors were delighted by the experience as advisor Prasit Bhattacharya would comment that he was "... impressed by the level of depth and understanding the students were able to attain in one summer ..." and participant Ethan Zell would also remark that he "... enjoyed the camaraderie of the [summer program] and felt very productive."



Cedric Harper Presents to Group

Reaching Out: Math Circles

UVA Mathematics Professor, Graduate Students Introduce New Program to Inspire Talented, Young Mathematicians

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The talented mathematicians sat hunched over their calculations as they worked through a series of problems on an unseasonably warm Sunday afternoon last December. Sunlight streamed through the windows of the University of Virginia's Charles L. Brown Science & Engineering Library, but no one, not even the small number cruncher wearing yellow soccer socks, shin guards and cleats, directed a wistful gaze outdoors as they wrestled with a triangular array of binomial coefficients known as Pascal's Triangle.

It was only when Professor Slava Krushkal called for a short break that the 14 assembled students reverted back to elementary-school form. A few remained in pairs at their tables, dueling over graph paper as they played extra rounds of the Shannon switching game of strategy introduced earlier by Krushkal, a mathematics professor in UVA's College and Graduate School of Arts & Sciences.

Math professor Slava Krushkal with 4th- and 5th-grade

The rest gravitated to the wheelchair ramp at the far corner of the room, giggling as they took turns hanging over the railings now serving as their own personal jungle gym. "Graduate students will just sit down and work for hours on a problem, but I've learned that with kids, you have to entertain them and take breaks," Krushkal says.

Continued on Page 10 —

Math Circle, continued

A different approach

Math-enrichment programs for interested young students are no longer uncommon in the United States. For the new Math Circle program that Krushkal launched last fall for 15 local 4th- and 5th-graders nominated by their schools, he looked for inspiration in his own educational experience growing up in Russia. As students achieved fluency in different mathematical concepts, the mathematics curriculum there stressed the importance of students honing their creativity as they think their way through increasingly difficult logic problems. The first Math Circles in the United States were organized by the Mathematical Sciences Research Institute in Berkeley, California. Today, there are more than 200 Math Circles around the country.

With the assistance of Gabriel Islambouli and Michael Reeks, two Ph.D. students in the Department of Mathematics, Krushkal organized a semester-long schedule of Sunday afternoon Math Circle sessions. Setting up the program, he received some guidance from the Curry School of Education, as well as from his faculty colleague and former Department of Mathematics chair, Craig Huneke.

"We have many first-rate mathematicians at UVA, and the establishment of Math Circles allows us to use this talent to help K-12 students in our area," Huneke said. "It is enormously empowering for children to think actively about the solution of mathematical problems, and to develop the solution themselves. No matter what the future holds for the children who go through Math Circles, the lessons they learn in problem-solving and self-reliance will be invaluable for them."

The goal of the program, Krushkal said, is to stoke the curiosity of young students who have displayed an early aptitude for mathematics, while steering away from the rote memorization that may accompany their traditional instruction.

"We are exposing them to logical problems, topology and other concepts that they may never see in school," Krushkal said. "Some of the things we show them are practical applications of math to real things. Some of them are fun mathematical games that feature deep mathematical principles underneath."

Earlier in the fall, Krushkal, Islambouli and Reeks introduced the students to "Hex," a strategic game devised independently in the 1940s by two mathematicians, Piet Hein and John Nash. On another Sunday, Krushkal dipped the frame of a triangular pyramid, built with sticks, into a soap solution. After he pulled the tetrahedron-shaped frame out of the solution, the bubble created within the frame came together at a single point smack in the middle of the open frame. That led to a discussion explaining how bubbles seek stability by minimizing their surface area. Soon, students were crafting other polyhedra out of sticks to make different bubbles.

"The bubbles are beautiful, and the students are saying, 'Wow!', and then we discuss the mathematical principles underlying the experiment as we explain how the soap bubbles try to minimize area as they achieve the ultimate shape," Krushkal said.

Islambouli said it was important for them to present the Math Circle students with interesting problems that not only challenge them, but also reward them for pursuing creative solutions.

"At first, of course, they were a little shy, he said. There's a lot of ego type stuff tied to math in school, so maybe they were afraid to be seen as dumb. But pretty soon, they realized, 'Oh, we're just playing games.' Once you rephrase it, and they realized they weren't being graded on their performance, it's no longer 'You versus your grade.' It's all of us trying to understand a problem together."

"It's really great to see some of the students really get into a problem and start arguing with us after we told them what the solution was," Reeks said. "Even if they were wrong, what really matters is that they had really taken ownership of the solution and the problem. One of the great things this program does is it exposes kids to different perspectives on what math is and how they can use quantitative reasoning in their lives. And I think that's helpful in heading off this symptom where people say, 'Oh I'm bad at math,' or 'I just don't like math' and close themselves off to that forever."

The UVA Math Circle took a break during the spring semester and was revived for the fall. Parents of the program's initial class of students said their children are eager to continue in the Math Circle next school year.

Michael Garcia said that the Math Circle program provided his son, a 5th-grade student at Walker Elementary, an opportunity to meet other students his age who share his interest in learning math.

"He was always eager to go and to share what he had learned," Garcia said. "I appreciated that he was given exposure to university faculty and students, introducing the idea that learning math is a long-term endeavor. The hands-on nature of the class really captured my son's attention."

Michele Paige Claibourn's daughter Fred (short for Winifred) came home from Math Circle eager to challenge her parents to play the strategic math games she learned in the Brown Science & Engineering Library.

"Maybe it's not the primary intent of the program, but one of the ways it's really helped Fred is the discovery of how fun it was to be in a room full of other students who also were really excited about math," Claibourn said. "That shared enthusiasm was new for her in some ways, and I think that as much as anything helped really reinforce to her that, 'This is cool, and here are some other people my age who also think this is cool.'"

The Math Circle program continued in Fall 2018. This year the program, taught by Slava Krushkal with the help of graduate students Peter Johnson and Rostislav Akhmechet, is geared towards 5-th and 6-th grade students from Charlottesville area schools.

Recent PhDs

John Berman

Categorified Algebra & Equivariant Homotopy Theory

Advisor Michael Hill

Since the mid-twentieth century, algebraic topologists and algebraic geometers have used categories to organize rich algebraic structure. However, during all this time there have been hints that categories are not just organizational tools, but important algebraic objects in their own right.

For example, algebraic invariants of a ring (including K-theory and Hochschild homology) are really invariants of the *derived* ∞*-category* Mod_{HR} of chain complexes of R-modules. This is the perspective of *noncommutative motives*, which have recently been developed by Blumberg, Gepner, Tabuada, and others [2]. The upshot is that noncommutative algebraic geometry should study *stable* ∞*-categories* [3], rather than rings or schemes.

Noncommutative algebraic geometry shows promise in approaching long-standing problems related to the field with one element. While $\mathbb{F}_1$ has no meaning by itself, it is meaningful to speak of an $\mathbb{F}_1$-module (of which the simplest examples are finite sets). This area is related to major open problems: the Riemann Hypothesis is an $\mathbb{F}_1$-analogue of the Weil Conjectures, and the Hodge Conjecture is an $\mathbb{F}_1$-analogue of the Tate Conjecture.

However, to study noncommutative motives over $\mathbb{F}_1$, we must study *algebraically* ∞-categories which are *not necessarily stable*. (There are also many other obstacles to studying $\mathbb{F}_1$ which we will not discuss.)

With this application in mind, the goal of my thesis is to introduce new purely algebraic techniques to study ∞-categories, building on [1]. This philosophy is best illustrated by two theorems and a conjecture (which may also function as definitions):

Theorem An additive ∞-category is a module over the Burnside ∞-category (as a semiring ∞-category).

Theorem A Lawvere theory is a cyclic module over Fin^{op} the opposite category of finite sets.

Conjecture An ∞-operad is a cyclic module over Fin^{op} which is trivializable over Burn.

Another potential application of these ideas is to equivariant homotopy theory. We will discuss the perspective that equivariance in homotopy theory arises from reparametrizing Lawvere theories over | finite G-sets. Finally, we introduce the following conjecture. Although we cannot define all the terms in the conjecture, we do state and prove a weaker version.

Conjecture The ∞-categories of *naive* and *genuine* equivariant spaces are dual as noncommutative motives over $\mathbb{F}_1$.

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[1] J. Berman. On the commutative algebra of categories. Submitted.

[2] A. Blumberg, D. Gepner, and G.Tabunada. A universal characterization of higher algebraic K-theory. Geom. Topol. 17: 733-838 (2013).

[3] J. Lurie. Higher Algebra. http://www.math.harvard.edu/lurie/ (2011). Preprint.

C*-algebras and other operator algebras are typically infinite-dimensional; yet some have enough finite-dimensional representations to recover their structure. A C*-algebra with enough finite-dimensional representations is called residually finite-dimensional. My research studies characterizations and applications of this and related properties, often in connection with universal objects and lifting arguments.

Definition A C*-algebra is Residually Finite-Dimensional (RFD) if it has a family of finite-dimensional representations whose direct sum forms an injection.

Before the property can be used, one must be able to identify whether a given C*-algebra is RFD. The following is a new characterization of residual finite-dimensionality in terms of the density of a distinguished subset of the C*-algebra.

Theorem (Courtney-Shulman) A C*-algebra is RFD iff it contains a dense subset of elements that attain their norm under some finite-dimensional representation.

Moreover, we can also characterize when this subset is the entire set.

Theorem (Courtney-Shulman) For any C*-algebra A, the following are equivalent.

1. Every element in A attains its norm under some finite-dimensional representation.
2. Every irreducible representation of A is finite-dimensional.
3. The set of elements of A that attain their norm under a finite-dimensional representation is an algebra.

Residual finite-dimensionality can yield surprising results in surprisingly elegant ways. For example, we can use it to generalize a famous inequality of von Neumann to certain noncommutative polynomials in two variables.

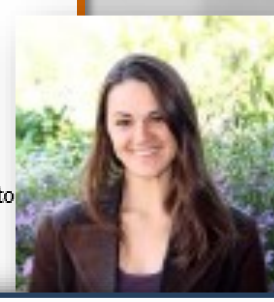
Theorem (Courtney) For any noncommutative polynomial p in two variables,

$\sup\{\|p(T, T^*)\| : T \text{ a contractive Hilbert space operator}\} = \sup\{\|p(T, T^*)\| : T \text{ a nilpotent matrix}\}$.

Moreover, we can connect this result to an infamous open conjecture of Alain Connes.

Theorem (Courtney) The following conjectures are equivalent.

1. Connes' Embedding Conjecture has a positive solution.
2. The universal unital C*-algebra generated by a pair of doubly commuting contractions is RFD.
3. The universal unital C*-algebra generated by a pair of doubly commuting contractions is RFD, and the separating family of representations can be chosen so that the generators map to nilpotent matrices.



Kristin Courtney

C*-algebras and finite-dimensional representations

Advisor David Sherman

Eloísa Grifo

Symbolic Powers and the Containment Problem

Advisor **Craig Huneke**

Hilbert's Nullstellensatz establishes a fundamental dictionary between geometry and algebra: the vanishing set of each radical ideal I in $\mathbb{C}[x_1, \ldots, x_n]$ is an algebraic subvariety of $\mathbb{C}^n$ and this correspondence is bijective. By the classical Zariski--Nagata Theorem, the polynomials that vanish up to order n along the variety defined by I coincide with the n-th symbolic power of I. Symbolic powers arise naturally from the theory of primary decomposition.

Definition Let P be a prime ideal in a domain R. The n-th symbolic power of P is the ideal given by

$$P^{(n)} = \{f \in R \mid gf \in P^n \text{ for some } g \notin P\} = \left\{f \in R : \frac{f}{g} \in P^n R_P \text{ for some } g \notin P\right\}.$$

This can be generalized to a definition for any radical ideal I.

It is natural to compare $I^{(n)}$ with the ordinary powers $I^{(n)}$. Even though $I^n \subseteq I^{(n)}$ always holds, we often have $I^n \neq I^{(n)}$. The Containment Problem asks when $I^{(a)} \subseteq I^b$. Over any regular ring, Ein-Lazarsfeld-Smith, Hochster-Huneke and Ma-Schwede showed that $I^{(hn)} \subseteq I^n$ for all $n \geqslant 1$, where h is an invariant of I known as the big height. Geometrically, the big height measures the largest codimension of an irreducible component of I.

Conjecture (Harbourne) If I is a radical ideal of big height h in a regular ring, $I^{(hn-h+1)} \subseteq I^n$ for all n.

Dumnicki, Szemberg and Tutaj-Gasińska showed that this fails for certain point configurations in $\mathbb{P}^2$. However, Harbourne's Conjecture does hold for nice classes of ideals, such as those corresponding to points in general position in $\mathbb{P}^2$ and $\mathbb{P}^3$, or for squarefree monomial ideals.

Theorem (Grifo-Huneke) Let R be a regular ring of characteristic $p>0$ (respectively, essentially of finite type over a field of characteristic 0), and let I be an ideal in R. If R/I is F-pure (respectively, of dense F-pure type), then I verifies Harbourne's Conjecture.

The class of F-pure rings includes determinantal rings, Veronese rings and "nice" rings of invariants. These examples are all in fact strongly F-regular (respectively, have log-terminal singularities), and if we restrict to this subclass, this result can be improved, replacing h by h-1. If h=2, this gives $I^n = I^{(n)}$.

Harbourne's Conjecture is as a generalization of an open question of Huneke: given a prime ideal P of height 2, must $P^{(3)} \subseteq P^2$? We show that this holds for the defining ideal of $k[t^a, t^b, t^c]$ in $k[x, y, z]$.

There is also evidence that Harbourne's Conjecture might always hold for all $n \gg 0$.

Theorem (Grifo) Let I be a radical ideal of big height h in a regular ring containing a field. If $I^{(hm-h)} \subseteq I^m$ for some $m \geqslant 2$, then $I^{(hn-h)} \subseteq I^n$ for all $n \gg 0$.

Christina Dawn Osborne

Decomposing the classifying diagram in terms of classifying spaces of groups

Advisor **Julie Bergner**

A topological space can be built from a category using the machinery of the nerve, which takes objects in the category to points and chains of n-composable morphisms to n-cells. The resulting space is referred to as the "classifying space" of the category. In particular, a group G is a category with one object; BG denotes its classifying space. However, two categories that are not equivalent can produce equivalent classifying spaces. The classifying diagram, defined in [2], is a generalization of the nerve. Two categories are equivalent if and only if their classifying diagrams are equivalent [1].

Definition [2] The *classifying diagram* of a category $\mathcal{C}$ is denoted by $N\mathcal{C}$ and is the simplicial space defined levelwise by

$$(N\mathcal{C})_n := \text{nerve}(\text{iso}(\mathcal{C}^{[n]})).$$

The purpose of this thesis is to provide a deeper understanding of the classifying diagram by providing levelwise characterizations using classifying spaces of groups. Let $\langle x \rangle$ denote the isomorphism class for $x \in \text{ob}(\mathcal{C})$. It was already shown that $N(\mathcal{C})_0 \simeq \coprod_{\langle x \rangle} BAut(x)$ [1]. We provide a new characterization of the higher levels of the classifying diagram.

Notation Let $\underline{x} = (x_0, \ldots, x_n)$ denote an $(n+1)$-tuple of objects in $\mathcal{C}$. We define a group action

$$\bullet : Aut(\underline{x}) \times \text{Hom}(\underline{x}) \longrightarrow \text{Hom}(\underline{x})$$

where $Aut(\underline{x})$ denotes the group $Aut(x_0) \times Aut(x_1) \times \cdots Aut(x_n)$ and $\text{Hom}(\underline{x})$ denotes the set $\text{Hom}_{\mathcal{C}}(x_0, x_1) \times \text{Hom}_{\mathcal{C}}(x_1, x_2) \times \cdots \times \text{Hom}_{\mathcal{C}}(x_{n-1}, x_n)$

Theorem Let $\langle f_1, \ldots, f_n \rangle$ denote the equivalence class of $(f_1, \ldots, f_n) \in \text{Hom}(\underline{x})$ defined by the group action $\bullet$. Then for $n \geq 1$,

$$N(\mathcal{C})_n \simeq \coprod_{\langle f_1, \ldots, f_n \rangle} B\left[Aut(\underline{x})_{(f_1, \ldots, f_n)}\right]$$

where $Aut(\underline{x})_{(f_1, \ldots, f_n)}$ is the stabilizer of $(f_1, \ldots, f_n)$.

We use the above result and prove explicit decompositions of the classifying diagrams for the categories of finite ordered sets, finite dimensional vector spaces, and finite sets.

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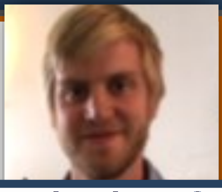

Michael Reeks

Trace and center of the twisted Heisenberg category

Advisor Weiqiang Wang

Categorification is the process of enriching an algebraic object by increasing its categorical dimension by one, e.g. passing from a set to a category or from a category to a 2-category. The original object can be recovered through the inverse process of decategorification. For example, the natural numbers $\mathbb{N}$ and the integers $\mathbb{Z}$ are both categorified by $\mathrm{Vect}_{\mathbb{C}}$, the category of finite-dimensional complex vector spaces, and the decategorification maps are taking the dimension dim and the Grothendieck group K_0, respectively.

Many important objects in representation theory have interesting categorifications, and the idea of studying an object and its representation theory by passing to a cateforification has proved to be fundamental. Most famously, Khovanov-Lauda and Rouquier independently constructed a 2-category which categorified the quantum group associated to a semisimple Lie algebra $\mathfrak{g}$. Cyclotomic versions of this 2-category categorify highest weight integrable modules of the quantum group.

The most common method of decategorification is taking the Grothendieck group – for example, the Khovanov-Lauda-Rouquier 2-category was constructed so that its Grothendieck group is isomorphic to the idempotent form of a quantum group. However, applying alternative decategorification functors can reveal additional structure encoded by categorifications. Applying one such functor, the trace, to the KLR 2-category gives the current algebra $\mathcal{U}(\mathfrak{g}[t])$, and taking the trace of the cyclotomic versions yields important representations of $\mathcal{U}(\mathfrak{g}[t])$ called local and global Weyl modules.

Another important example of a Categorification is Khovanov's Heisenberg category, which is rich in representation theoretic and algebra-combinatorial information. Its Grothendieck group contains (and is conjecturally equal to) the Heisenberg algebra. However, the trace contains much more structure: it is isomorphic to the Lie algebra of differential operators on the circle, $W_{1+\infty}$, at level one. This is a large and rich algebra which contains the Heisenberg algebra. Another decategorification functor, the center, gives an algebra of shifted symmetric functions with connections to the asymptotic representation theory of symmetric groups.

In this dissertation, we investigate the trace and center of a twisted version of the Heisenberg category, denoted $\mathcal{H}_{tw}$, which was introduced by Cautis and Sussan to categorify the twisted Heisenberg algebra. We establish a connection between the even trace of $\mathcal{H}_{tw}$ and a classical type of subalgebra W^- of $W_{1+\infty}$ (which itself is a type A construction related to $\mathfrak{gl}_\infty$) introduced by Kac, Wang, and Yan.

Theorem The even trace of $\mathcal{H}_{tw}$ is isomorphic as an algebra to W^- at level one.

The center of $\mathcal{H}_{tw}$ is a polynomial algebra generated by closed curl diagrams, called bubbles. We show that this is in fact a subalgebra of the symmetric functions Λ.

Theorem The center of $\mathcal{H}_{tw}$ is isomorphic to the subalgebra $\Gamma \subset \Lambda$ generated by $\{p_{2n+1} | n \in \mathbb{N}\}$, where p_r is the power-sum symmetric function.

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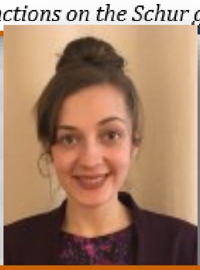


Veronica Shalotenko

In Search of Bounds on the Dimension of Ext between Irreducible Modules for Finite Groups of Lie Type

Advisor Brian Parshall

Let q be a power of a prime p, and let $G(q)$ be a finite group of Lie type defined over $\mathbb{F}_q$. Let k be an algebraically closed field of characteristic r. There are three distinct cases to consider in the representation theory of $kG(q)$: $r = 0$ (the characteristic 0 case), $r = p$ (the defining characteristic case), and $r > 0$, $r \neq p$ (the non-defining characteristic, or cross-characteristic, case).

In defining and non-defining characteristic, we are interested in finding bounds on the dimension of cohomology groups $H^i(G(q), V)$, where V is any irreducible $kG(q)$-module and $i \geq 1$ is fixed. In the defining characteristic, such bounds exist when the rank is fixed (this is due to Cline, Parshall, and Scott in the $i = 1$ case and Bendel, Nakano, Parshall, Pillen, Scott, and Stewart in the $i > 1$ case). In 2011, Guralnick and Tiep showed that 1-cohomology groups are bounded in non-defining characteristic.

Theorem (Guralnick and Tiep) Suppose that $G(q)$ is a finite simple group of Lie type and $\mathrm{char}(k) = r > 0$ with $r \neq p$. Let W be the Weyl group of $G(q)$, and let e be the Lie rank of $G(q)$. If V is an irreducible $kG(q)$-module, then

$$\dim H^1(G(q), V) \leq \begin{cases} |W| + e & \text{if } V \text{ is in the unipotent principal series,} \\ 1 & \text{otherwise.} \end{cases}$$

Some Generalizations of Guralnick and Tiep's Theorem

Suppose that $G(q)$ has a split BN-pair of characteristic p and that we are in the non-defining characteristic case (so, $\mathrm{char}(k) = r > 0$ with $r \neq p$). We use techniques of modular Harish-Chandra theory to obtain the following bounds on the dimension of Ext^1 between irreducible $kG(q)$-modules.

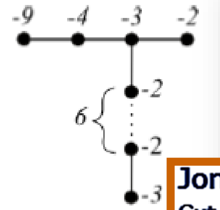
Theorem (Shalotenko) If Y and V are irreducible $kG(q)$-modules in the unipotent principal series, then $\dim \mathrm{Ext}^1_{kG(q)}(Y, V) \leq |W| + \min(\dim Y, \dim V)e$.

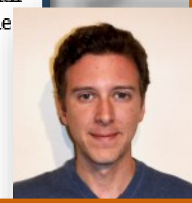
Theorem (Shalotenko) Suppose that $G(q) = \mathrm{GL}_n(q)$, $\mathrm{SO}_n(q)$ with n and q odd, or $\mathrm{Sp}_n(q)$ with n even and q a power of 2. Let $\mathrm{char}(k) = r$ be a linear prime for $G(q)$. If Y is a unipotent principal series representation and V is an irreducible outside the unipotent principal series, then $\dim \mathrm{Ext}^1_{kG(q)}(Y, V) \leq [W : W_J]$, where W_J is a parabolic subgroup of W defined using the Harish-Chandra vertex of V.

Ext Groups between Irreducible $k\mathrm{GL}_n(q)$ Modules

We also consider higher Ext groups in the case that $G(q) = \mathrm{GL}_n(q)$. In 1999, Cline, Parshall, and Scott related H^i calculations for $\mathrm{GL}_n(q)$ (in non-defining characteristic) to Ext^i calculations over a q-Schur algebra. In this thesis, we generalize the results of CPS to relate certain Ext^i calculations over $\mathrm{GL}_n(q)$ (in non-defining characteristic) to Ext^i calculations over $S_q(n, n)$ (which, in turn, translate to Ext^i calculations over an appropriate quantum group).

Since Milnor discovered the first pairs of manifolds that are homeomorphic but not diffeomorphic in 1956, simply connected smooth manifolds have been classified by purely algebraic tools in every dimension except dimension 4. The goal of this research is to introduce a new technique of constructing exotic 4-manifolds via the following cut-and-paste process: (1) locate a configuration of 2-spheres C embedded in an ambient 4-manifold; (2) cut out a neighborhood of C called a *plumbing*, which can be described by a weighted graph; and (3) glue in its place a 4-manifold with smaller homology. This technique generalizes the *rational blow-down*, an operation that has been successfully used to construct exotic 4-manifolds.





Jonathan Simone

Cut-and-paste operations and exotic 4-manifolds

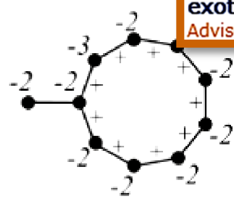
Advisor Thomas Mark

We begin with a heuristic definition.

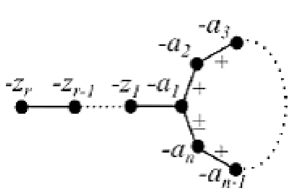
Definition A plumbing is k-replaceable if it can be "symplectically replaced" by a manifold with Euler characteristic k.

1-replaceable plumbings are precisely those that can be rationally blown down. Which plumbings are 2-replaceable, you ask? Using Lisca's classification of symplectic fillings of lens spaces and the theory of Lefschetz fibrations, we classify 2-replaceable linear plumbings and construct 2-replaceable plumbing trees. We then locate the 2-replaceable plumbing depicted to the right in a blowup if $E(1)$ and "replace it" with its Euler characteristic 2 symplectic replacement to obtain a 4-manifold X. Using Freedman's Theorem and Seibert-Witten invariants, we prove the following.

Theorem X is a symplectic exotic $\mathbb{C}P^2\#6\overline{\mathbb{C}P}^2$.



With 1- and 2- replaceable plumbings understood, we explore the possible existence of 0-replaceable plumbings (this exploration is still underway). First, we construct families of plumbings that can be smoothly replaced by rational homology $S^1 \times D^3$s. Using such plumbings, it is routine to construct manifolds that are homeomorphic to well-known 4-manifolds. For example, using the plumbing depicted to the right, we can construct a 4-manifold that is homeomorphic to, but not obviously diffeomorphic to, $\mathbb{C}P^2\#\overline{\mathbb{C}P}^2$. This would be the first example of an exotic $\mathbb{C}P^2\#\overline{\mathbb{C}P}^2$. To help determine whether these manifolds are indeed exotic, we can: (a) compute smooth 4-manifold invariants; or (b) determine whether these manifolds are symplectic. As a step towards (a), we prove a result that computes the Ozsváth-Szabó 4-manifold invariant under certain conditions. Unfortunately, this theorem cannot be used to prove the particular example above is an exotic $\mathbb{C}P^2\#\overline{\mathbb{C}P}^2$ because the "certain conditions" are not satisfied. As a first step in exploring (b), we classify the tight contact structures with no Giroux torsion on the boundaries of the family of plumbings depicted to the left. This classification involves proving the following results, which is a generalization of an important result due to Lisca and Matic.



Theorem Suppose (Y,ξ) is a contact 3-manifold with nontrivial contact invariant $c(\xi,[\omega])$, for some $[\omega] \in H^2(Y;\mathbb{R})$. Let (W,J_i) be a Stein cobordism from (Y,ξ) to $(\tilde{Y},\xi_i)$, for $i = 1, 2$. If the spin^c structures induced by J_1 and J_2 are not isomorphic, then ξ_1 and ξ_2 are nonisotopic tight contact structures.

Zachary Gates
[Advisor: Peter Abramenko]
Finite presentability of groups acting on locally finite twin buildings

Bradley Weaver
[Advisor: Andrew Obus]
Non-abelian groups of order eight and the local lifting problem

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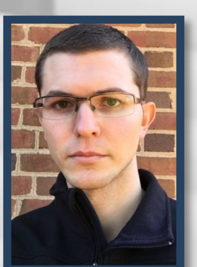

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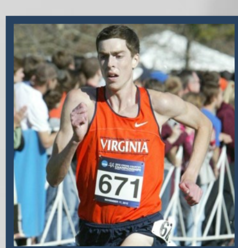


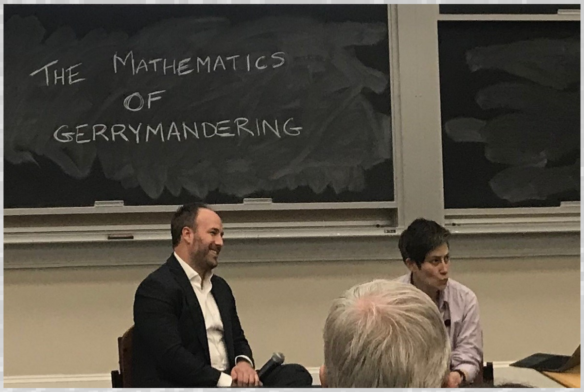
Zachary Gates

Moon Duchin Speaks about Mathematics and Gerrymandering

By Sara Maloni

On March 21-22-23, 2018, Moon Duchin visited us, as the last speaker of the series "Math &...". Moon Duchin is an associate professor of Mathematics, a senior fellow in the Jonathan M. Tisch College of Civic Life, and the director of the Program in Science, Technology, and Society at Tufts University. Her pure math work is in geometry, topology, groups, and dynamics, and she is currently collaborating with civil rights organizers, coders, political scientists, lawyers, geographers, and philosophers on a large-scale project to detect and address gerrymandering.

On Thursday she gave a public talk titled "Mathematics of Gerrymandering: Geometry, Algorithms, and (Crimes Against) Democracy". After the talk, Moon was joined on stage by Brian Cannon (from OneVirginia2021) and Sally Hudson (from Frank Batten School of Leadership and Public Policy) who moderated the discussion between Moon and Brian. The event was very well attended. I think for all the participants from the Math Department it was exciting to see such interesting and important applications to problems we study in our research. On Friday, Moon joined Quantitative Collaborative and gave a talk titled "Data-Driven Approaches to Identifying a Gerrymander."



In her talk Moon pointed out that the power to draw boundary lines is often the power to control election outcomes, and hence the key question she wants to study is: how much skew is too much, and when should the courts step in? Recently, some interventions from mathematics and computing have sprung on the redistricting scene, and they seem to hold a lot of promise. For example, at the beginning of the year the Pennsylvania Governor Tom Wolf asked Moon to aid him in coming up with nonpartisan Congressional district maps. Moon surveyed some geometric, statistical, and algorithmic approaches to identifying and addressing gerrymandering that have gotten traction in Wisconsin, North Carolina, and Pennsylvania. In particular, she explained how she used algorithmic sampling methods to explore the space of possible districting plans.

This visit was made possible by the generous help from the Arts & Science Diversity and Inclusion Grant, the Department of Mathematics, Department of Politics, Frank Batten School of Leadership and Public Policy, OneVirginia2021, and Quantitative Collaborative.

The Problem Corner

1. Find all differentiable functions f(x) on the real line such that $[f'(x)]^2 = 4f(x)$ and $f(-2) = f(2) = 1$.

2. A microbe either splits into two perfect copies of itself or else disintegrates. If the probability of splitting is p, what is the probability that one microbe will produce an everlasting colony?

3. A game of solitaire is played as follows. After each play, according to the outcome, the player receives either a or b points, where a and b are positive integers and $a > b$. Their score accumulates from play to play. It is noticed that there are exactly 35 non-attainable scores and that one of those is 58. Find a and b.

4. You are given 13 integers, all of them at least 1. They have the property that if we remove any one of them, you can divide the remaining 12 into two groups of 6 each such that the sum of those 6 are equal. Prove all thirteen numbers are the same.

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